Multiobjective Optimization of Molded LDPE Foams Characteristics Using Genetic Algorithm

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ABSTRACT: This article concerns the injection manufacturing process of molded foam sheets and their intrinsic shock and noise performances. The main goal is to optimize the physical performances of molded plastic foams at an early stage in their design and manufacturing. The effects of injection process parameters on the properties of molded LDPE foams are investigated. The input optimization parameters considered are as follows: injection temperature, mold temperature, injection speed, plasticization back pressure, and screw rotation speed during the plasticization phase. The output optimization parameters considered are as follows: density, shock absorption, and acoustic absorption. The experimental design method made use of the central composition design. This allows us to identify simplified mathematical models for input/output and to detect the most influential input in the injection process. Ultimately, models are used to carry out multiobjective optimization of injected foams characteristics in the presence of a few constraints on decision variables. This optimization is done using a very robust technique, NSGA-II. Several two-objective functions involving sometimes the maximization and other times minimization of foam characteristics have been studied to illustrate the procedures and explain and interpret the results obtained. One needs to solve several simpler optimization problems with just one or two decision variables (smaller amount of freedom), to gain insight and to provide help in formulating the more general multiobjective optimization problem.
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Key words: blowing agents; injection molding; mechanical properties; polyethylene; processing

INTRODUCTION

Plastic foams are made up of scattered cavities. These cavities enhance lightness and softness, while adding to the soundproofing and heatproofing qualities. Polymer foams are increasingly used in industrial applications. Made up of a structure of more or less regular open or closed cells, they are expected to have a high energy absorption capacity (particularly useful for shock applications), efficient acoustic and thermal insulating properties, and in some cases interesting filtering properties. For these reasons, they are widely used in the aircraft industry, automobiles, buildings, packaging, etc. Combining good mechanical properties with low density, rigid polymer foams can also be used as structural materials. Whatever their use, optimization requires an understanding of the relationship between their processing and mechanical properties. This will be the focus of this article.

There are numerous techniques for plastic foam processing and they vary according to the final product application. Among the most common industrial processing techniques are extrusion and injection. Extrusion is very often used and is easier to perform than injection. This is mainly due to the fact that there are fewer process parameters to consider for manufacturing optimization purposes. In contrast, the injection process implies a greater number of parameters having a direct impact on the final product aspect and properties. Injection molding machines commonly consist of three parts. The plasticization unit is used to melt the blend and inject it through the nozzle into the mold cavity. The second part is the mold and its cavity. And finally, the closing unit is the part which clamps the mold. To obtain a good foam plastic piece, certain injection conditions have to be applied. First, the plasticizing cylinder must remain permanently attached to the fixed part of mold so that the injection barrel nozzle never leaves the mold nozzle during the injection cycle. On the other hand, it is not recommended to maintain pressure so as to obtain a natural expansion of the material. The solidification running time must be large and necessary to rigidify the walls of the foam cavities and completely squeeze the gas. The next condition, in the plasticizing stage, consists of preparing the exact volume to fill up the mold cavity. This stage begins just after the injection phase

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(amount of materiel injected was determined by sequential injection to obtain a complete piece, this is for every design of experiment configuration). As for the direct feed nozzle, i.e., without injection threshold nor feeding canals, the injection point is directed at the center of the part. The ejection of the specimen is done manually to minimize the specimen deformation that would occur if ejector pins pushed the specimen out of the cavity. To ensure a rigorous study, the first 10 molded parts were eliminated and the effects of changed parameters were observed after every five molded pieces.

Many studies of the relationship between the plastic foam extrusion process and the resulting mechanical properties of the foam can be found in the available literature.[1–4] The injection process of compact plastic materials is studied in references.[5,6] But very few articles consider the foam injection process.[7] This article aims to give insight into the injection processing-mechanical properties of LDPE foams.

The effect of extrusion conditions on the physicochemical and mechanical properties of plastic foams has been covered in many articles. Working with expanded rice snacks, Ding et al.[1] for instance, investigated the effect of extrusion conditions including: feed rate, feed moisture content, screw speed, and barrel temperature on the physicochemical properties (density, expansion, water absorption index), on the water solubility index (WSI), and on sensory characteristics (hardness and crispness) of the expanded rice snack. The authors indicated that a higher barrel temperature increased the extrudate expansion but reduced density, while increasing the WSI and crispness of the extrudate. Screw speed had no significant effect on the physicochemical properties and sensory characteristics of the extrudate. Jeong and Toledo[2] studied the effects of CO_2 injection pressure on the expansion ratio, bulk density, porosity, water absorption index, WSI, specific mechanical energy, average cell size, cell area ratio, and cell density. An experimental investigation was carried out to study the rheological behavior of ethylene propylene diene rubber compounds in extrusion containing a blowing agent.[3] The cell morphology development and rheological properties were studied for gum and carbon black-filled systems with variation of the blowing agent, extrusion temperature, and shear rate. An analysis of cell morphology of high-density polymer foams was studied by Grosselin and Rodrigue.[4] This study proposes methods to calculate the surface cell count as well as approaches for converting surface cell in volume cell.

Taking advantage of the experimental design method, Postawa and Koszkul[5] studied the influence of processing conditions (mold temperature, injection temperature, cooling time, hold pressure, and injection speed) for selected properties that characterize the injection molded piece, such as mass, longitudinal shrinkage, transverse shrinkage, and diversity of the processing shrinkage within the confines of one molded piece. The authors used central composition design to describe the investigated properties of the molded pieces as a function of the most significant input parameters. Nagaoka et al.[6] studied dependencies between characteristics of the core layer resin, and the skin/core ratio in particular with the injection molding conditions. In their article, they consider the influences that the molding conditions such as injection speed, cylinder temperature, and mold temperature confer on the mechanical properties of sandwich moldings. It was demonstrated that the core cylinder temperature and mold temperature could be used to adjust the mechanical properties of sandwich injection moldings. In the case of single material sandwich moldings, the injection speed seemed to play no significant role, even though it was clearly demonstrated that the core volume increases with injection speed.

Concerning foam injection studies, an article written by Lee and Cha[7] should be mentioned. The thickness of the skin layer on parts made with a microcellular injection molding process may influence its properties, including impact and tensile strength, density, and sound transmission. Lee and Cha studied the variations in skin layer thickness with processing parameters, particularly the mold temperature. Yuan and Turng[8] studied microstructures and mechanical properties of microcellular injection-molded polyamide-6 nanocomposites. Cell wall structure and smoothness were determined by the size of the crystalline structure, which, in turn, were based on the material system and molding conditions. Yuan and Turng studied exponential correlation between cell density and cell size of the materials studied. On the other hand, Chedly et al.[9] selected injection process parameters that may influence on foams characteristics. Authors selected five parameters among 11: injection temperature, mold injection, back pressure, injection speed, and plasticizing speed. Density distribution over injected plates and bulk material quantity (that filled cavity on injection machine) were studied by authors,[10] and it concluded that distribution is parabolic where the top is the injection point. Polynomials models of density distribution, according to process parameters, were established, and Chedly et al.[11] studied robustness of these models. Recently, a new article studied process parameters impact on shock and acoustic absorption. Chedly et al. took into account foams injection process. Studied parameters are as follows: injection temperature, mold temperature, injection speed, plasticization back pressure, and screw rotation speed during the plasticization phase. Experiments were done on foams to evaluate the shock and acoustic absorption characteristics. Shock and acoustic models were built using design of experiments. And for validation, tests were done on specimens manufactured with central values of process parameters. Finally, models robustness was analyzed using Taguchi principle.

In this article, the relationship between the plastic foam injection process and the resulting mechanical properties of the foam are considered. The input optimization parameters considered are as follows: injection temperature, mold temperature, injection speed, plasticization back pressure, and screw rotation speed during the plasticization phase. The main outputs considered for the optimization are as follows: density, shock absorption, and acoustic absorption. The experimental design method was implemented using the central composition design. This allows us to identify simplified mathematical models for input/output and to detect the most influential input in the injection process. Ultimately, models are used to carry out multiobjective optimization of injected foams characteristics in the presence of a few constraints on decision variables. This optimization is done using a very robust technique, NSGA.

MATERIALS AND PROCEDURES

In this section, the materials, equipment, and procedures employed are briefly described. First, the materials used for foam manufacturing are presented as well as their characteristics. Then, the equipment and injection conditions concerning plastic foam production are described. Experiment design theories are the object of the third part of this section, where a brief review of the central composition design is presented. Finally, input/output factors used in the experimental design method are defined including testing principles and the optimization strategy.

Materials

Plastic foams are used when a proportion of cavities filled by gas are required in the structure of a polymer matrix. The matrix is a low-density polyethylene Lupolen 2420 H, purchased from Basell Company. The blowing agent master batch is Palmarole BA F4 E, purchased from Adeka Palmarole. This master batch contains $\sim$ 40% of a chemical endothermic blowing agent (sodium hydrogen carbonate) dispersed in the LDPE matrix. A master batch blend of LDPE and blowing agent was employed. These blends contain 6% Palmarole BA F4 E master batch (mechanical pellet blends, prepared in a turbomixer with rotational speed blades at 350 rpm, for 15 min).

Equipment

Injection molding machines commonly consist of three parts. The plasticization unit is used to melt the blend and inject it through the nozzle into the mold cavity. The second part is the mold and its cavity. And finally, the closing unit is the part which clamps the mold. Samples of investigations have been made using injection method by means of Billion H260/470. It is not recommended to maintain pressure so as to obtain a natural expansion of the material. The solidification running time was defined during tests and set to 200 s. Specimens manufactured were 150 $\times$ 150 mm^2 forms, 16-mm thick, and the injection point was in the center of the part.

Brief review of the experimental design method

During the injection process of compact plastic material pieces, many parameters can affect the overall product quality. In our case, a polymer matrix blend with a blowing agent was used and under the effect of heat, a chemical transformation occurred, releasing gas. And so, a first hypothesis can be given: some process parameters may have an effect on foam characteristics. To clarify this point, experiment design theories were used as the quantification and qualification technique for understanding the effects of injection process parameters on plastic foam product characteristics. The problem may be assimilated to the black box, which presents an input/output system. Inputs $(X_1, X_2,\ldots, X_n)$ are the process factors (design parameters, controlled variables) and outputs $(Y_1, Y_2,\ldots, Y_m)$ are the measurable responses (product characteristics). This system can be disturbed by noise factors (ambient temperature, process sensitivity...). The main objective is to find a relationship between influential inputs and the outputs with a prediction error, ε, minimizing noise factors:

$$Y_i = F(X_1, X_2,..., X_n) + \varepsilon \quad 1 \le i \le m \qquad (1)$$

Central composition design is a technique based on statistics in which a complete factorial design on two levels is supplemented by a central point and two additional points for each factor (called the "star points"). Thus, five levels are defined for each factor. The central point and star points are added to obtain information on the design space within and beyond the two levels of complete factorial design, which makes it possible to estimate factor effects at higher orders. In short, complementary tests are represented on each factor axis by points located at a distance α of the field center. The entire set of tests includes: N_F factorial design tests 2^k, N_0 central tests, and N_A star tests. Central composition designs are not orthogonal, so one can seek a quasi-orthogonality by increasing the number of tests in the center. The value of α depends on a criterion of optimality: the criterion characterizing central composition design.

$$\alpha = (N_F)^{1/4} \tag{2}$$

This table makes it possible to express experimental data in sophisticated models: linear model with interactions or a polynomial model with or without interactions. Model general form has the following expression:

$$Y = a_0 + \sum_i a_i X_i + \sum_{i,j} b_{ij} X_i X_j + \sum_j c_j X_j^2 \tag{3}$$

Where, Y is the studied output, X_i the input value, a_i, b_{ij}, and c_j the model coefficients, and a_0 is the model constant.

Input/output factors in the experimental design method

The injection molding process requires the adjustment of several parameters. For this study, five factors were considered and are referred to as "inputs" in this article (with respect to the purpose of optimization): A, defines the injection temperature; B, the mold temperature; C, the hydraulic back pressure (defined as the pressure applied during the screw rotation stage, it improves dissolving of the blowing agent in LDPE matrix); D, the injection speed; and E, the screw rotation speed during the plasticizing stage. By modifying one or more of these process parameters, the molten material state changes and thus results in a different microstructure and different properties of the injected foam. The output parameters considered concern some of the foam mechanical properties. The average mass density of the specimen was considered and was calculated from 15 samples of the same batch. The shock absorption coefficient was the second output considered and established using shock tests. The final physical parameter considered is the acoustic absorption coefficient.

Shock tests[11] were conducted with a machine, which comprises an impact system (with a force and a displacement device), and a data acquisition and processing system. Instrumentation in force, P, and in displacement, l, of the test gives directly shows the evolution of P according to l. By integration this curve, we find the evolution of energy, E, throughout the test. Typical curves $E(l)$ are presented later (Fig. 1). These variations allow to determine two

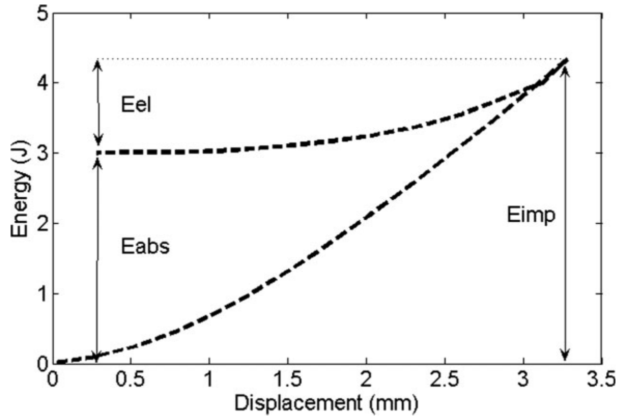

Figure 1 Impact energy against displacement (LDPE, e = 8 mm, H = 292 mm, and m = 2300 g).

characteristic values of the test: the energy absorbed by the material, E_{abs}, it is presented on the curve $E(l)$ by the final value of the relief zone; and the real impact energy delivered to the material, E_{imp}. Tested specimens were 100 × 100 mm^2, 8-mm thick. They were cut from injected specimens. These samples are skinless (a 4-mm layer was removed on each side with a micromilling machine). For each specimen, a shock test was carried out with a constant altitude (H = 142 mm) and a constant mass (m = 2300 g). From curve $E(l)$, the absorbed energy value, which appear as important to characterize injected foams, was defined to compute output shock coefficient α_{shock}:

$$\alpha_{\mathrm{shock}} = \frac{E_{\mathrm{abs}}}{E_{\mathrm{imp}}}, 0 \le \alpha_{\mathrm{shock}} \le 1 \tag{4}$$

The acoustic absorption coefficient defines the third output parameter, and the stationary wave tube[11] is the main tool used to determine material acoustic absorption characteristics. The specimen was attached at one end of the tube, and a loudspeaker at the other extremity. In the center of this last, there is a hole through which a microphone support can be inserted. The exact position of the microphone is located with a graduated ruler. Excitation and measure strings include essentially the acquisition, treatment, and synthesis digital units (Siglab) connected to computer. Tested specimens were disc-shaped 100 mm in diameter and 8-mm thick. They were manufactured from injected foam samples using a micromilling machine. These specimens were foam, skinless discs. The acoustic absorption coefficient curve (Fig. 2) cannot be fitted by a mathematical equation, hence, the choice of 30 values corresponding to 30 definite frequencies. These values were defined as output parameters. Specimens used in acoustic absorption tests were taken to

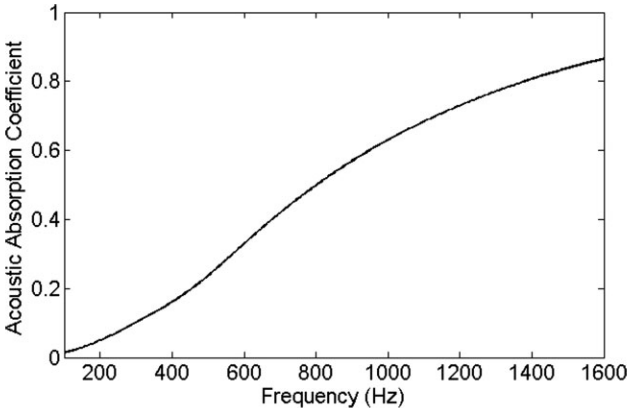

Figure 2 Acoustic absorption coefficient according to frequency.

measure foam densities; the latter is defined as the last mechanical property, i.e., the output parameter.

SHOCK AND ACOUSTIC MODELS

Searching appropriate models representing correctly outputs according to factors in the design space is the aim of this part. The central composition table was used to find models, which would fit with experimental data. The aptitude of models to predict outputs according to given factors was verified by carrying out experimental tests, setting the input factors at centered values.

In this study case, the number of design factors $n = 5$. Therefore, the number of factorial design tests $N_F = 32$ tests are normally required. However, using fractional design, only $N_F = 16$ tests are needed and the new factor level $\alpha = 2$. The number of star point tests $N_A = 10$ and of centered point tests $N_0 = 10$ (orthogonal hypothesis of the central composition design).The total number of tests to be performed is thus 36 tests.[11]

A, B, C, D, and E are the factors used in the investigation. The lower and upper limits for each factor are −2 and +2, respectively. These limits take into account the injection machine performance limit and the achievement of a good sample. Level values −1, 0, and +1 were obtained by linear extrapolation allowed. In the table later (Table I), those different values for each factor are presented:

On the basis of the gathered investigation results, statistical analyses have been performed using Lumiere software®, which allows the model coefficients for each investigated output to be extracted, with 95% confidence.

The polynomial model without interactions $(Y = a_0 + \sum_i a_i X_i + \sum_j c_j X_j^2)$ appears as the better fitting experimental data. To verify the aptitude of the model to predict outputs according to given factors, experimental tests were performed, setting the input factors at the centered values ($T_{\text{inj}} = T_m = \text{CP} = V_{inj} = V_{dos} = 0$). Then, a comparison was made of experimental results Y_{exp} and theoretical results Y_{theo}, which were obtained from model equation set on the centered values, to check if Y_{theo} is a number in the interval $\lfloor Y_{\text{exp}} - 2\sigma, Y_{\text{exp}} + 2\sigma \rfloor$, where σ is the experimental standard deviation. When the latter condition is verified, the theoretical model is considered to fit with experimental data. If not, the theoretical model is considered not to fit and then a more sophisticated model should be investigated.

For example, the appropriate model considered to fit experimental data for the case of foam density is polynomial without interactions. This output depends on injection temperature, back pressure, injection speed, and screw rotation speed. The model has the following format:

$$d_m = 0.1587 + 0.001 \times T_{\text{inj}} - 0.0059 \times \text{CP} + 0.0012 \times V_{\text{inj}} + 0.0075 \times \text{CP}^2 + 0.0127 \times V_{\text{dos}}^2 \quad (5)$$

The a_i and c_c coefficient values obtained enable us to determine the investigated output value at any point of the design space limited by the input parameters. High values of the received adjustment coefficients (greater than 0.9) prove that the model equations properly describe (with a small deviation) the dependence between the investigated output values and the input processing parameters.

d_m increases with V_{inj}, indeed a rapid filling of the mold allows rapid cooling and thus gives a higher value to d_m. On the other hand, increasing T_{inj} causes an increase in foam density. Increasing the dosing speed and back pressure causes, at first, when $-2 \leq \text{CP}, V_{\text{dos}} \leq 0$, the decrease of d_m and, when $0 \leq \text{CP}, V_{\text{dos}} \leq +2$, it causes d_m to increase. Indeed, when the back pressure increases, gas dissolved better in the mattress, so it expands in the mold and foam density decreases. According to this model, dosing speed and then back pressure have the most effects on foam density because of higher values of theirs coefficients. Therefore, expansion phenomenon is happen in majority during dosing step. It seems that, on the other hand, these two process parameters favor crash and acoustic absorption coefficients. And so, increasing foam density induces

TABLE I
Factors Values at Different Levels

		−2	−1	0	+1	+2
A	T_{inj}	200	212.5	225	237.5	250
B	T_m	20	27.5	35	42.5	50
C	CP	10	15	20	25	30
D	V_{inj}	90	117.5	145	172.5	200
E	V_{dos}	90	115	140	165	190

TABLE II
Model Correspondence for Central Values

	Theoretical value	Experimental value
d	0.3484	[0.3403–0.351]
d_m	0.1587	[0.148–0.169]
α_{Shock}	0.5705	[0.569–0.572]
α_{acous} (954 Hz)	0.1442	[0.138–0.159]
α_{acous} (1330 Hz)	0.1931	[0.185–0.202]

amelioration of foams shock and acoustic characteristics:

$$\alpha_{Shock} = 0.5658 - 0.0126 \times T_m + 0.0141 \times CP^2 + 0.0171 \times (V_{dos} + 0.5234)^2 \quad (6)$$

$$\alpha_{acous}(f = 954Hz) = 0.1047 + 0.0161 \times T_m + 0.0203 \times V_{dos}^2 + 0.0035 \times (T_{inj} = 3.2428)^2 + 0.0203 \times (V_{inj} + 0.3645)^2 \quad (7)$$

Increasing injection speed induces crushing cells on mold wall and so the increase of void on the mold, increasing material quantity injected and ultimately foam density. The same phenomenon is observed in mass density model:

$$d = 0.3491 - 0.0101 \times T_{inj} - 0.0048 \times T_m - 0.013 \times CP^2 - 0.0132 \times (V_{inj} - 0.231)^2 \quad (8)$$

With regards to data in Table II, the polynomial model does correctly represent outputs according to factors in the design space. In the case of inputs central values, theoretical crash absorption was 0.5705. This value is a number of the interval [0.569–0.572]; interval defines experimental values of shock absorption performed with inputs centered values. Ultimately, the polynomial model is considered as fitting with experimental data for the case of α_{shock}.

On the other hand, acoustic absorption coefficients models have same form and same influents factors as seen in eqs. (7) and (9).

$$\alpha_{acous}(f = 1330Hz) = 0.1931 + 0.0299 \times T_{inj} + 0.0388 \times T_m + 0.0192 \times V_{inj} + 0.0284 \times T_{inj}^2 + 0.0277 \times V_{inj}^2 + 0.0227 \times V_{dos}^2 \quad (9)$$

MULTIOBJECTIVE OPTIMIZATION

In this work, mathematical models are first developed for injected foams mechanical characteristics (global density, foam density, shock, and acoustic absorption coefficients). The experimental results from experiment design are used to provide estimates of variables and parameters required in models. Thereafter, multiobjective Pareto-optimal solutions are generated for use in the manufacturing and characterization of injected foams using genetic algorithm (GA).

GA is a nontraditional search and optimization method,[12–14] which has become quite popular in engineering optimization. It mimics the principles of genetics and the Darwinian principle of natural selection (i.e., survival of the fittest). Simple genetic algorithm (SGA) is suitable for optimizing problems with a single objective function. In single objective function optimization, one attempts to find the best design, which is usually the global minimum (or maximum). Most real world problems involve the simultaneous optimization of multiple objective functions (a vector). Such problems are conceptually different from single objective function problems. In multiple objective function optimization, there may not exist a solution that is the best (global optimum) with respect to all objectives. Instead, there could exist an entire set of optimal solutions that are equally good. These solutions are known as Pareto-optimal (or nondominated) solutions. A Pareto set, for example, for a two-objective function problem is described by a set of points such that when one moves from one point to any other, one objective function improves, whereas the other worsens. Thus, one cannot say that any one of these points is superior (or dominant) to any other. Because none of the nondominated solutions in the Pareto set is superior to any other, any one of them is an acceptable solution. The choice of one solution over the other requires additional knowledge of the problem, and often, this knowledge is intuitive and nonquantifiable. The Pareto set, however, is extremely useful because it narrows down the choices and helps to guide a decision maker in selecting a desired operating point (called the preferred solution) from among the (restricted) set of Pareto-optimal points, rather than from a much larger number of possibilities.

Several methods are available to solve multiobjective optimization problems, e.g., the constraint method,[15–17] goal attainment method,[18–20] and the nondominated sorting genetic algorithm (NSGA).[13,21,22]

The algorithm devised in this study is based on the concept of nondominated sorting originally conceptualized by Goldberg[13] and developed by Srinivas and Deb[21] as the NSGA. The NSGA is not an entirely new optimization algorithm, but rather a modification to the fitness evaluation procedures that exist in standard GAs. It is in some sense a supplement to a GA that allows for a more effective means of multiobjective optimization. This technique (NSGA) offers several advantages.[14] For instance, its efficiency is relatively insensitive to the shape of the Pareto-optimal front, problems with uncertainties, stochasticities, and with discrete search spaces can

TABLE III
Objective Functions

Objective function 1	Objective function 2
Minimize d_m	Minimize d
Minimize d_m	Maximize α_{shock}
Minimize d_m	Maximize α_{acous} (954 Hz)
Minimize d_m	Maximize α_{acous} (1330 Hz)
Maximize α_{shock}	Maximize α_{acous} (954 Hz)
Maximize α_{shock}	Maximize α_{acous} (1330 Hz)

be handled efficiently. On the other hand, the "spread" of the Pareto set obtained is excellent (in contrast, the efficiency of other optimization methods decides the spread of the solutions obtained), and finally, it involves a single application to obtain the entire Pareto set (in contrast to other methods, e.g., the constraint method, which needs to be applied several times over). Indeed, NSGA has been applied recently to optimize several processes of industrial importance in chemical engineering, including an industrial Nylon-6 semibatch reactor,[22,23] a wiped-film polyester reactor,[24] a steam reformer,[25] and cyclone separators.[26]

Multiobjective optimization of injected LDPE foam characteristics

NSGA was used with the tuned model described earlier to optimize the mechanical characteristics of injected LDPE foams. Multiobjective Pareto-optimal solutions are generated here for use in both the operating phases and characteristics of plastic foams. To ensure that physically meaningful results are obtained, constraints on process parameters (as injection temperature, mold temperature, back pressure, injection speed, and dosing speed) were added to our model. These constraints are necessary to ensure that process parameters stay within reasonable limits assumed by injection equipments.

Therefore, problem, that is multiobjective optimization one with constraints on parameters, may be formulated as:

$$\text{Minimize}\, F(x) = \{f_1(x), f_2(x)\} \tag{10}$$

Such as $x_r^{\min} \leq x_r \leq x_r^{\max}$, $1 \leq r \leq n$.

In this study, objective functions may be as follows: mass density, foam density, shock absorption coefficient, and acoustic absorption coefficients at 954 and 1330 Hz. Multiobjective optimization depends of characteristics target: maximization or minimization of the two functions or minimize one and maximize the other. In the table later (Table III), the choice of objective functions is presented. To reduce material cost, it is legitimate to minimize mass and foam densities. To ameliorate vibroacoustic performances, maximize shock and acoustic absorption coefficients are the ideal choice.

As the computer code available with us for NSGA minimizes the objective functions, we need to transform the objective function to maximize into one involving minimization. Several candidates are available for this, but probably the simplest and the most popular form (which also does not change the location of the solutions) is to minimize ($-f_2$), rather than maximize f_2. Thus, an example of the optimization problem studied (minimize d_m and maximize α_{acous} (954 Hz)) is represented mathematically by:

$$\begin{aligned}\text{Min } d_m \times \text{Max } \alpha_{acous}(954Hz) = \text{Min}\{-\alpha_{acous}(954Hz)\}\\ \times \text{Such that} -2 \leq A, B, C, D, E \leq +2\end{aligned} \tag{11}$$

The decision variables for this problem are taken as (i) injection temperature, A; (ii) mold temperature, B; (iii) back pressure, C; (iv) injection speed, D; and (v) dosing speed, E. These choices would usually be available at the design stage. The bounds of the decision variables used are tabulated in Table I. Variation area of these objective functions must be between 0 and 1; condition verified when parameters fluctuate between -2 and $+2$.

Figure 3 presents point's sets corresponding to different typical solutions: they present different arrangements between the limitation of foam density and mass density. Pareto set is obtained with a population of 200 and 100 generations, parameters fixed such as the achievement of a continued set. It can easily be confirmed, because Pareto fronts, that as the shock absorption coefficient increases (desirable), the acoustic absorption coefficient decreases (undesirable) and the foam density increases (undesirable) and as the foam density increases (undesirable), the mass density decreases (desirable) and the acoustic absorption coefficient increases (desirable). To validate these sets, one point has selecting from every set and using process parameters corresponding to

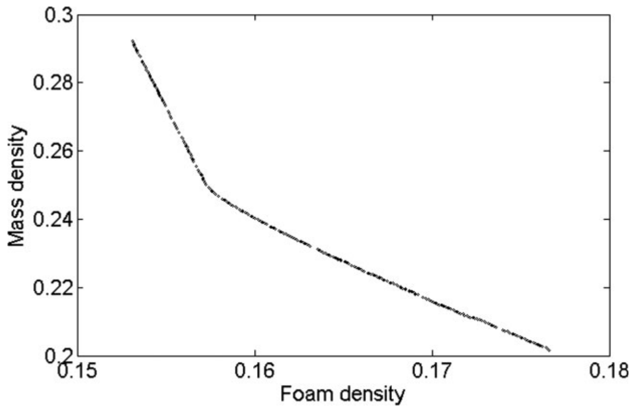

Figure 3 Pareto set for foam density and mass density.

TABLE IV
Optimal Pareto Sets Validation

Pareto set (f_1, f_2)	$f_{1\text{theo}}$	$f_{2\text{theo}}$	Process parameters					$f_{1\text{exp}}$	$f_{2\text{exp}}$
			T_{inj}	T_m	CP	V_{inj}	V_{dos}		
(d_m, d)	0.1766	0.2016	+2	+2	+2	−2	−0.0885	[0.1746–0.1788]	[0.1998–0.2024]
(d_m, α_{Shock})	0.1948	0.7034	−0.7263	−2	+2	−2	1.2868	[0.1946–0.2003]	[0.7016–0.7085]
(d_m, α_{acous} (954 Hz))	0.2127	0.4278	+2	+2	0.3935	+2	+2	[0.2106–0.2188]	[0.4246–0.4288]
(d_m, α_{acous} (1330 Hz))	0.2143	0.6741	+2	+2	0.8588	+2	+2	[0.2112–0.2168]	[0.6736–0.6764]
α_{Shock}, α_{acous} (954 Hz))	0.7234	0.4054	+2	0.61	+2	+2	+2	[0.7214–0.7237]	[0.4011–0.406]
α_{Shock}, α_{acous} (1330 Hz))	0.7059	0.6741	+2	+2	+2	+2	+2	[0.6992–0.712]	[0.6728–0.6758]

these points (Table IV), specimens were manufacturing and foams characteristics were defining.

The table mentioned earlier presents also experiments results of specimens manufacturing to validate Pareto sets. It is clearly that theoretical characteristics are belong to interval experimental ones, these last are obtained using parameters process corresponding to particular points chosen randomly. For instance, Pareto curve (d_m, d (Fig. 3) is the points set of minimum foam density and shock absorption coefficient. To obtain an injected foam with a mass density equal to 0.2016 and a foam density equal to 0.1766, process parameters are follower: an injection temperature equal to 250°C, a mold temperature equal to 50°C, a back pressure equal to 30 bars, injection speed equal to 90 cm^3/s, and a dosing speed equal to 138 rd/min. Using these parameters, measurement of mass and foam densities leads to an interval of values that theoretical characteristics are belong. In this case, experimental foam density fluctuates between 0.1746 and 0.1788 and mass density is between 0.1998 and 0.2024.

Dominant decision variable

The Pareto-optimal set $\alpha_{\text{Shock}}-\alpha_{\text{acous}}$(1330 Hz)), problem no.1, is shown in Figure 4(a), while the five decision variables, according to the shock absorption coefficient, corresponding to the points on the Pareto set are shown in Figure 4(b), respectively. It is found that the optimal values of four of these five decision variables are almost constant (T_{inj}, CP^2 (CP at their upper bound for $0.706 \leq \alpha_{\text{shock}} \leq 00.722$ and their lower bound for $0.722 \leq \alpha_{\text{shock}} \leq 00.756$), V_{inj}, and V_{dos} at their upper bounds), and only T_m decreases as the values of α_{Shock} increases and as the values of α_{acous}(1330 Hz) decreases.

In fact, in the latter case, T_m emerges as the single dominant decision variable, varying along the Pareto-optimal points, while all the other four decision variables take on constant values either at their upper or lower bounds. A similar phenomenon of dominating decision variables was observed in a multiobjective optimization study of trains of cyclone separators.[26] This is a manifestation of the sensitivity of the objective functions to the decision variables.

To validate these results (that T_m is the single dominant decision variable), multiobjective optimization is used to present Pareto-optimal set of $\alpha_{\text{Shock}} - \alpha_{\text{acous}}$(1330 Hz)), where $T_{\text{inj}} = V_{\text{inj}} = V_{\text{dos}} = 2$ and $\text{CP}^2 = 4$. Curves superposition proves that the two Pareto sets, where the first with the five decision variables and the other with a dominant decision variable, are the same. Curves showing the shock absorption coefficient and the five decision variables according to the acoustic absorption coefficient give the same conclusions. This procedure may be applied to the other Pareto sets of our study case.

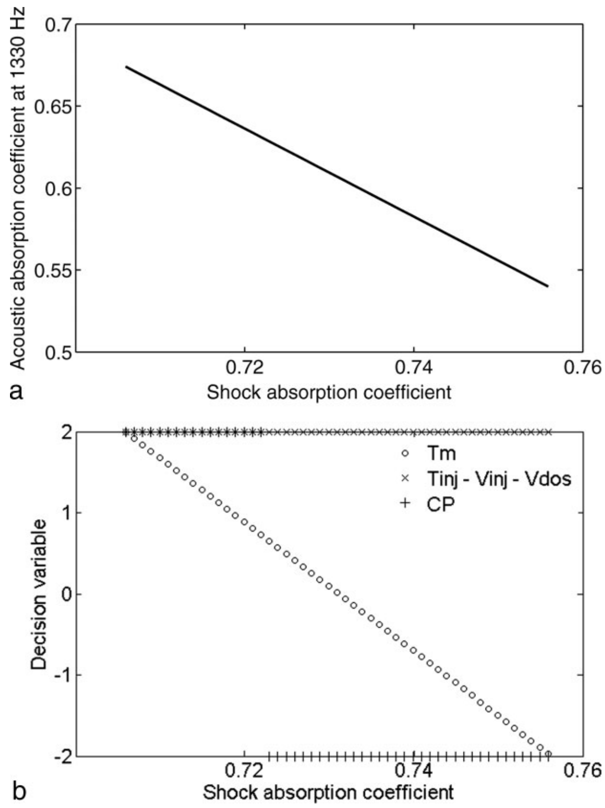

Figure 4 (a and b) Pareto set and values of the decision variables corresponding to the points on the Pareto set.

TABLE V
Description of Parameter Values Used to Study Problem Nos. 1–10

	Problem no. 1	Problem no. 2	Problem no. 3	4	Problem no. 5	6	Problem no. 7	8	Problem no. 9	10
T_{inj}	−2..+2	+2	+1	0	+2	+2	+2	+2	+2	+2
T_m	−2..+2	−2..+2	−2..+2	−2..+2	−2..+2	−2..+2	−2..+2	−2..+2	−2..+2	−2..+2
CP^2	0..+4	+4	+4	+4	2	0	+4	+4	+4	+4
V_{inj}	−2..+2	+2	+2	+2	+2	+2	+1	0	+2	+2
V_{dos}	−2..+2	+2	+2	+2	+2	+2	+2	+2	+1	0

Sensitivity of the pareto set to (constant) values of T_{inj}, CP, V_{inj}, and V_{dos}

As discussed in the previous section, the mold temperature, T_m, is the most important decision variable, which controls the optimal solution. It is, therefore, natural to use T_m as the sole decision variable, and fix the others (T_m, CP^2, V_{inj}, and V_{dos}) at constant values. This would lead to less cumbersome design procedures. This is precisely what has been done in problem no. 2 (Table V), in which T_{inj}, CP^2, V_{inj}, and V_{dos} have been kept constant. The Pareto set obtained is almost identical to that for problem no. 1 [Fig. 4(a)], which is not surprising, because the constant values of T_{inj}, CP^2, V_{inj}, and V_{dos} were taken to be almost the same as those obtained in problem no. 1. Problem no. 2 is being called as the reference (ref) case.

A sensitivity study is now carried out. The values of T_{inj}, CP^2, V_{inj}, and V_{dos} are changed one at a time, and optimal solutions (with T_m as the sole decision variable) obtained. Table V shows details of the several (problem nos. 3–10) studied, and Figures 5 and 6 show the results graphically. These diagrams are useful to an engineer manufacturing injected LDPE foams and will enable him, after targeting a particular foam characteristic, to select feasible (and optimal) process parameters. These diagrams quantify the trade-offs available. Figure 6 allows one to know the kind of trade-offs that can be made between T_m and one of the other four decision variables, T_{inj}, CP, V_{inj}, and V_{dos}. This is useful for the case when an engineer does not want to make more than two alterations to an injection unit, as for example, if an engineer must working with a higher mold temperature.

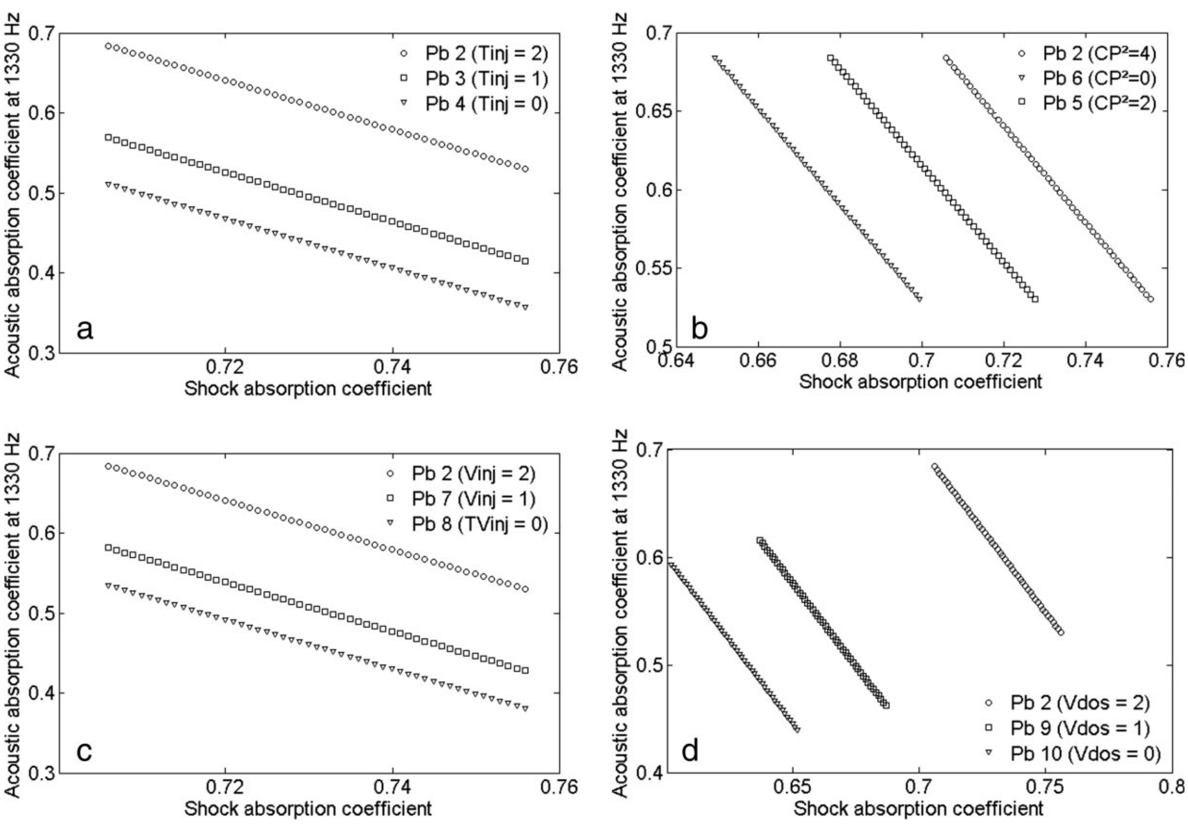

Figure 5 Effect of (a) T_{inj}, (b) CP, (c) V_{inj}, and (d) V_{dos} on the Pareto sets. T_m is the only decision variables used.

From the top chart of Figure 6, he would find out that he has to decrease the injection temperature to achieve the previous acoustic and shock absorption coefficients, α_{Shock} and α_{acous}(1330 Hz).

Pareto area

The aim of this part is to define the region where values of two objective functions are possible. The idea is based on applying the NSGA in different cases possible of optimization: minimize f_1 and f_2, maximize f_1 and f_2, minimize f_1 and maximize f_2, and maximize f_1 and minimize f_2.

Optimal Pareto sets obtained are superposed to form an area of objective functions values: Pareto area. To validate this result, experimental data (used in design of experiment stage) were confronted to Pareto area. Figure 7 presents confrontation between experimental data and Pareto area for (d_m, α_{Shock}) objective functions, it is clear that experimental points have surrounded by Pareto sets. These curves give an idea to the engineer about shock and acoustic performances of an injected LDPE foam with taking into account limit performances of injection machine and material characteristics material used. For example, with process parameters fluctuating between −2 and +2, an engineer cannot manufacture an injected foam that has a mass density equal to 0.2, a foam density equal to 0.16, a shock absorption coefficient equal to 0.5, and an acoustic absorption coefficients equal to 0.05 and 0.1 at 954 and 1330 Hz, respectively.

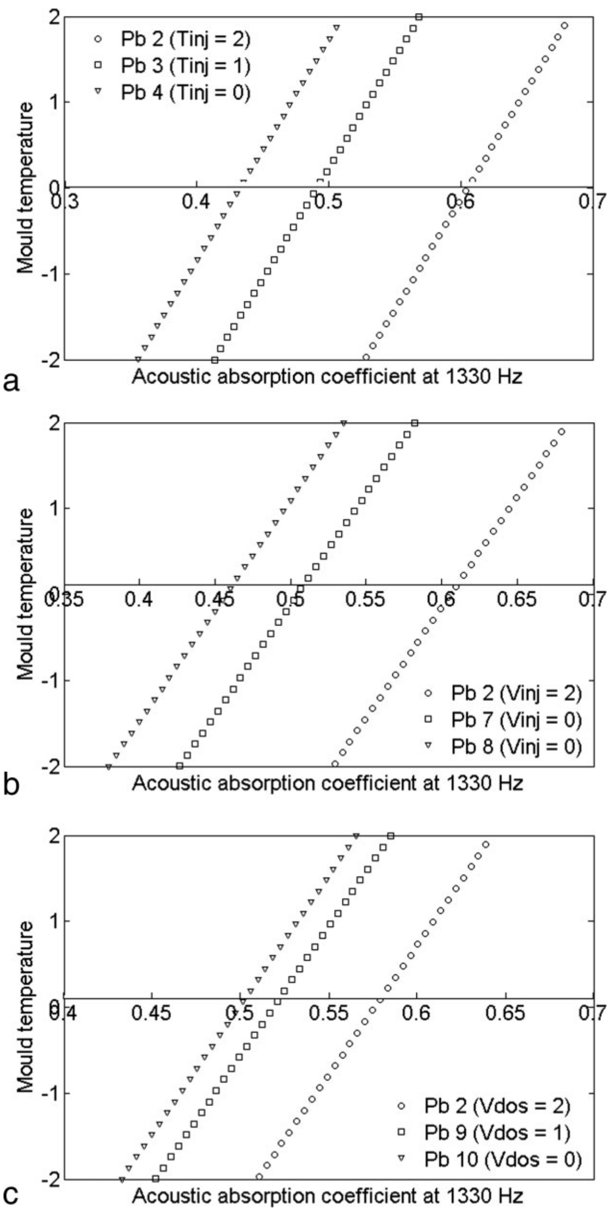

Figure 6 Effect of (a) T_{inj}, (b) V_{inj}, and (c) V_{dos} on the optimal value of T_m corresponding to the Paretos.

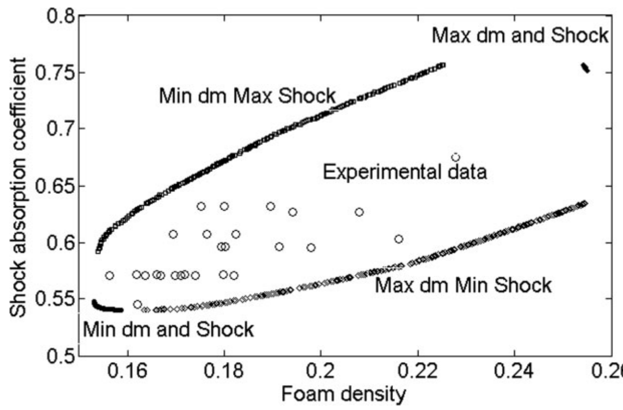

Figure 7 Pareto area.

CONCLUSIONS

Robust models expressing some injected LDPE foams characteristics according to some process parameters have been developed in this article, and the use of experiment design theories proved to be a good tool to found linear models fitting experimental data.

The input parameters considered are as follows: injection temperature, mold temperature, injection speed, plasticization back pressure, and screw rotation speed during the plasticization phase. The main outputs considered for the optimization are as follows: density, shock absorption, and acoustic absorption. The choice of an appropriate model was done by realizing confirmation tests on central values of inputs. In the last part of this article, models are used to carry out multiobjective optimization of injected foams characteristics in the presence of a few constraints on decision variables. This optimization is done using a very robust technique, NSGA. Several two-objective functions involving sometimes the maximization and other times minimization of foam characteristics have been studied to illustrate the procedures and explain and interpret the results obtained. One needs to solve several simpler optimization problems with just one or two decision variables (smaller amount of freedom), to gain insight and to provide help in formulating the more general multiobjective optimization problem.

The method used for investigation seems to be a good one for the optimization of all plastics materials injection process. Nevertheless, the model is function of the material nature, specimen shape, and the injection machine. Therefore, to make a correct use of it and to correlate the presented results with those of another injection product, it seems to be necessary to perform comparative investigations.

NOMENCLATURE

$X = (X_1, X_2, \ldots, X_k)$	System inputs
$Y = (Y_1, Y_2, \ldots, Y_m)$	System outputs
F	Function input/output
ε	Prediction error
n	Factors number
$-\alpha$, -1, 0, $+1$, and $+\alpha$	Factor levels $\alpha = 2$)
N_F	Factorial design test number
N_A	Central test number
N_0	Star test number
P	Force (Shock test)
l	Displacement (Shock test)
E	Energy (Shock test)
t	time (Shock test)
e	specimen thickness
H	Impact altitude
m	Impact mass
g	Gravity
l_{max}	Maximal displacement
E_{el}	Elastic energy
E_{imp}	impact energy
E_{abs}	Absorbed energy
α_{shock}	Shock absorption coefficient
$\alpha_{acous}(f)$	Acoustic absorption coefficient
f	Frequency
d	Mass density
d_m	Foam density
T_{inj}	Injection temperature
T_m	Mold temperature
CP	Back pressure
V_{inj}	Injection speed
V_{dos}	Screw rotation speed
a_0	Model constant
a_i	1st order Input coefficient
c_j	2nd order Input coefficient
b_{ij}	Interactions coefficient
Y_{theo}	Theoretical output value
Y_{exp}	Experimental output value
σ	Experimental standard deviation

References

1. Ding, Q. B.; Ainsworth, P.; Tucker, G.; Marson, H. J Food Eng 2005, 66, 283.
2. Jeong, H. S.; Toledo, R. T. J Food Eng 2004, 63, 425.
3. Guriya, K. C.; Bhattachariya, A. K.; Tripalty, D. K. Polymer 1998, 39, 109.
4. Grosselin, R.; Rodrigue, D. Polym Test 2005, 24, 1027.
5. Postawa, P.; Koszkul, J. J Mater Process Technol 2005, 162/163, 109.
6. Nagaoka, T.; Ishiaku, U. S.; Tomari, T.; Hamada, H.; Takashima, S. Polym Test 2005, 24, 1062.
7. Lee, J. J.; Cha, S. W. Cell Polym 2005, 24, 279.
8. Yuan, M. Y.; Turng, L.-S. Polymer 2005, 46, 7273.
9. Chedly, S.; Chettah, A.; Onteniente, J. P.; Bonnefoy, H.; Ichchou, M. In Proceedings of ECCM; Biarritz, France, 2006.
10. Chedly, S.; Chettah, A.; Onteniente, J. P.; Bonnefoy, H. In Proceedings of Macro 2006; Rio de Janeiro, Brazil, 2006.
11. Chedly, S.; Chettah, A.; Bonnefoy, H.; Ichchou, M. Polym Eng Sci 2008, 48, 12.
12. Holland, J. H. Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control and Artificial Intelligence; University of Michigan Press: Ann Arbor, 1975.
13. Goldberg, D. E. Genetic Algorithms in Search, Optimization, and Machine Learning; Addison-Wesley: Reading, MA, 1989.
14. Deb, K. Optimization for Engineering Design: Algorithms and Examples; Prentice Hall: New Delhi, India, 1995.
15. Chankong, V.; Haimes, Y. Y. Multiobjective Decision Making–Theory and Methodology; Elsevier: New York, 1983.
16. Haimes, Y. Y. Hierarchical Analysis of Water Resources Systems: Modeling and Optimization of Large-Scale Systems; McGraw-Hill: New York, 1977.
17. Hollingdale, S. H.; Lighthill, J. Methods of Operational Analysis in Newer Uses of Mathematics; Penguin: New York, 1978.
18. Fonseca, C. M.; Fleming, P. J. IEEE Trans Syst Man Cybernet A 1998, 28, 26.
19. Fleming, P. J. In Proceedings of IFAC Control Applications of Nonlinear Program and Optimization; Capri, Italy, 1985; p 47.
20. Fleming, P. J. Electron Lett 1986, 22, 258.
21. Srinivas, N.; Deb, K. Evol Comput 1995, 2, 106.
22. Mitra, K.; Deb, K.; Gupta, S. K. J Appl Polym Sci 1998, 1, 69.
23. Gupta, R. R.; Gupta, S. K. J Appl Polym Sci 1999, 73, 729.
24. Bhaskar, V.; Gupta, S. K.; Ray, A. K. AIChE J 2000, 46, 1046.
25. Rajesh, J. K.; Gupta, S. K.; Rangaiah, G. P.; Ray, A. K. Ind Eng Chem Res 2000, 39, 706.
26. Ravi, G.; Gupta, S. K.; Ray, M. B. Ind Eng Chem Res 2003, 78, 2.